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Research article

Estimating Portfolio Risk Measures: The contribution of GoF Tests

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Abstract

This paper uses Goodness-of-Fit (GoF) tests to select the true marginal distributions and the true copula function that requires portfolio risk measure estimation. Best model contains Skew Student margins and Student copula function. Comparison with different other models show that dependence structure has a smaller effect on risk measure than marginal distributions.

Keywords: Copula, GoF techniques, dependence structure, risk measures, financial risk, Monte Carlo simulation.

1. Introduction

Internal models using VaR must take into account empirical correlations between risk factors. These empirical correlations can be measured bu copula approach. A risk measure that not introduce empirical links can mislead a financial institution to error. A risk measure must respect axioms[1] proposed by Artzner et al. (1999)to characterize a coherent risk measure. Estimating VaR suffers from estimation risk, model risk and risk definition.

Measuring dependence structure effect on risk measure is studied by Cheng, Li & Shi (2007). Combined effect of dependence strcuture and marginals is studied by Ané & Kharoubi (2003), Junker & May (20065) and Fantazzinni (2009). Our contribution in this paper consists using GoF techniques based on Kendall tau to select best copula and best marginals in a multivariate case. Paper is structured as follows: second section presents data and methodology, while marginal estimation and selection will be presented in section three. Fourth section concentrates on copula estimation and selection and fifth section to present risk measure estimations. Sixth section concludes and discusses results.

2. Data and methodology

[1] sub-additivity, monotony, pertinence, positive homogeneity and invariant translation.

To estimate portfolio risk we use ten financial assets and a set of marginal distributions and different copulas.

2.1.Data description

Our data consists of ten assets from NYSE on a long period covering many financial events that can affect financial series volatility. Table 1 summarizes descriptive statistics of assets.

	AET	BA	C	DIS	F	GE	IBM	KO	UTX	XOM
Minimum	-22.69%	-19.38%	-49.43%	-34.23%	-28.77%	-19.22%	-26.82%	-28.14%	-30.28%	-26.65%
Maximum	25.22%	14.38%	45.63%	17.40%	25.87%	17.97%	12.36%	17.88%	12.79%	16.39%
Mean	0.04%	0.04%	0.02%	0.05%	0.05%	0.05%	0.04%	0.06%	0.05%	0.06%
Stdev	2.05%	1.98%	2.76%	2.02%	2.49%	1.80%	1.77%	1.62%	1.76%	1.54%
Skewness	-0.51	-0.17	-0.60	-0.79	0.00	-0.12	-0.38	-0.43	-0.67	-0.50
Kurtosis	14.35	5.61	42.41	18.52	12.45	8.49	13.01	17.47	14.99	19.66
JB***	67523.55	10291.36	586956.3	112640.1	50572.42	23532.59	55353.88	99799.97	73827.07	126339.3
p-values	0	0	0	0	0	0	0	0	0	0

Notes: Daily returns are calculated on daily adjusted closing prices from splits and dividends of 10 assets. data are collected from Yahoo.com. Minimum is the minimum loss. Maximum is the maximum gain. mean is the average return on period. std is the standard deviation. JB test is the Jarque Bera test and its probability is p-value. data is KO (The Coca Cola Company, Consumption), GE (General Electric, Energy), DIS (The Walt Disney Company, Leisure),AET (Aetna Inc., Insurance), XOM (Exxon Mobil, Energy), C (CitiGroup, Finance),UTX (United Technologies, Conglomerates), BA (Boeing, Aeronautic), F (Ford, Automobile Industry), IBM (International Business Machines, Technology).

Table 1: Descriptive statistics of assets on 1980-2010 period

From Table 1 we can see that financial series are not normally distributed regard to JB p-values and skewness and kurtosis values. In fact, risk measures based on Gaussian distribution can underestimate effective losses and this suggests using asymmetric distribution like Student or skew Student.

2.2. Methodology

In order to estimate portfolio risk measure using copula we follow de Mendes & de Souza (2004) methodology. The effect of modifying copula parameters or marginal distribution parameters on risk measure estimation is assessed by comparison between true model selected with GoF technique and other models. We use several marginal distributions and several copulas to reach our target. Table 2 gives an idea on types of copulas and marginal distributions to use in this paper.

	Symmetric		Asymmetric	
Marginal distribution	Normal	Student	Skew Normal	Skew Student
Copula	Normal	Student	Clayton	Gumbel

Notes: We use symmetric marginal distributions and copulas: Normal, Student. Asymmetric marginal distributions and copulas are: Skew Normal, Skew Student, Clayton copula (Clayon (1978)) and Gumbel copula (Gumbel (1958)). Models are constructed respect to Sklar's theorem and combined marginal distributions and copula to have a Joint Distribution Function of portfolio.

Table 2: Models used in estimating portfolio risk

Gaussian distribution is used to comparison purposes with classical models of risk measures. Student distribution allows to model financial series with high kurtosis. Asymmetric marginal distributions are useful for asymmetry

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characteristic of series. Skew Student marginal distribution is adopted by Patton (2004, 2006), Jondeau & Rockinger (2003), Fantazzinni (2009) to model financial series. Skew normal distribution captures asymmetry and complete standard normal distribution.

All these distributions are interconnected and can take into account stylized facts of financial series. Methodology steps are:

- Estimate and select best marginal distribution by GoF technique;
- Estimate and select best copula by GoF technique;
- Construct a Multivariate JDF and simulate data with Monte Carlo;
- Estimate VaR and CVaR risk measures and evaluate the impact of misspecification on risk measure estimation

2.3. VaR and CVaR risk measures

Let $\alpha \in [0,1]$ a given probability level and $\lambda \in \Delta_N$, VaR with probability α of return R_λ defined by:

$$VaR_\alpha(R_\lambda) = \inf\left(X \middle| P(R_\lambda \le X) \ge \alpha\right) = -F_{R_\lambda}{}^{-1}(\alpha) \qquad [1]$$

$F_{R_\lambda}^{-1}$ function is generalized inverse of cumulative distribution function $F_{R_\lambda}(x) = P[R_\lambda \le x]$ and gives α -quantile of R_λ . $VaR_\alpha(R_\lambda)$ is maximal potential loss that supports a portfolio $\lambda \in \Delta_N$ in $100(1-\alpha)\%$ cases, which means that which low probability α , portfolio return is less than $-VaR_\alpha(R_\lambda)$. VaR has many limits and CVaR is an alternative that respects a coherent risk measures axioms. CVaR is given by:

$$CVaR_\alpha(R_\lambda) = -\frac{1}{\alpha}\left\{E(R_\lambda 1_{R_\lambda \le x^\alpha}) - x^\alpha\left(P\left[R_\lambda \le x^\alpha\right] - \alpha\right)\right\} \qquad [2]$$

Where $x^\alpha = F_{R_\lambda}^{-1}(\alpha)$

3. Estimation and GoF of marginals

Financial series are characterized by asymmetry, fat tail distribution and departure from normality. We estimate marginal distributions presented in Table 2 by ML. Likelihood function $L(x_1, x_2, ..., x_n, \theta)$ is optimized with iterative methods. θ vector contains marginal distributions parameters that are:

- Mean μ and standard deviation σ for Normal distribution;
- Mean μ , standard deviation σ and DoF ν for Student distribution;
- Mean μ , standard deviation σ and skewness for Skew normal distribution;
- Mean μ , standard deviation σ , Skewness and DoF ν for Skew Student distribution.

To select which marginal distribution is the right for our data, we use GoF technique.

3.1.GoF test for marginals

Selecting marginal distribution is crucial for portfolio risk measure estimation. D'Agostino & Stephens (1986) gives a survey of techniques used to select best marginal distribution. In this paper we use Cramer-von-Mises (CvM) test with the null of Normal distribution. GoF test consist of repeat the test with Monte Carlo simulation approach, n times and compare test value with repetitions sorted in ascending order. If test value is in 5% first values, then we accept the null with reject probability of 5%. Otherwise we reject the null of Normal distribution. Table 3 summarizes percentages of rejection of the null.

H0: Normal distribution	AET	BA	C	DIS	F	GE	IBM	KO	UTX	XOM
Ha:Normal	4.94%	4.82%	4.77%	4.98%	5.22%	5.11%	4.90%	5.26%	5.25%	4.95%
Ha:Student	66.58%	14.67%	39.49%	86.74%	45.21%	50.05%	100%	81%	37.75%	72.67%
Ha:Skew Normal	4.74%	4.67%	5.28%	5.25%	5.04%	5.28%	4.71%	5.10%	4.91%	4.97%
Ha:Skew Student	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Notes: Percentage of rejection of the null of Normal distribution is based on CvM test and Monte Carlo simulation approach. Test is repeated 10000 times and values are sorted in ascending order. If the test value belongs to first 5% values then we accept the null. Otherwise we present the percentage of rejection.										

Table 3: Percentage of rejection of the null hypothesis of Normal distribution

We accept null hypothesis when using alternative hypothesis Normal or Student marginals due to low value of skewness. GoF test rejects null hypothesis with 100% when data alternative is Skew Student which proves that best marginal distribution is Skew Student for all financial series.

4. Estimation and GoF of Copula

We estimate copula parameters by inversion of Kendall tau approach of Genest, remillard & Quessay (2006) and select the best copula with GoF technique based on CvM test.

4.1. Multivariate copula

Copula[2], as founded by Sklar (1959), is a statistical tool having several advantages for modeling dependence structure between risk factors in finance. It offers flexibility in multivariate analysis, authorizes less restrictive joint distributions for financial series that capture stylized facts.

Copula theory permits decomposition of multivariate joint distribution in univariate margins and a dependence function which gives possibilities to extend many results obtained in univariate case to multivariate case. Multivariate joint distribution given by copula are very close to financial series reality. In non elliptical world, copula plays the same role of linear correlation in elliptical world.

We select useful copulas[3] presented in Table 2 having different dependence structures. Normal and Student copulas belong to elliptical family with symmetric dependence structure and Clayton and Gumbel copulas are from archimedean family with asymmetric dependence structure. But Gumbel copula is either archimedean and extreme value copula. Copulas parameters estimation is performed by inversion of Kendall tau approach of Genest et al. (2006). Given that Kendall tau is:

$$\tau = 4\int_{[0,1]} C_{u,v}\, dC_{u,v} - 1 \qquad [3]$$

[2] Best references of copula are Nelsen (1999), Cherubini, Luciano & Vecchiato (2005) and Joe (1997)
[3] Normal, Student, Clayton and Gumbel copulas formulas are given in Nelsen (1999)

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Let C_θ a copula family and to estimate the parameter θ we an analogy with moments method. We assume the relation $\theta = g(\theta)$ where g a continuous function. θ is estimated by :

$$\hat{\theta}_n = g^{-1}(\tau_n) \qquad [4]$$

4.2. GoF test of copula

Copula parameters estimations is not enough to say what is the copula that fits the data better. Information criterion, Akaike or Bayesian Information Criteria (AIC) and (BIC), are usually used to select models. These criteria don't give an information neither on the size nor on the power of decision taken which signifies that we can't say how selected copula family fits data. A GoF technique gives this information. Many studies have focused on copula GoF, we cite Malvergne & Sornette (2003), Fermanian (2005), Berg (2007), Genest et al. (2008) et Genest & Rémillard (2008).

To test if a copula under the null belongs or not to a given family, empirical copula is the very objective reference since it is totally non parametric. Consequently, a natural GoF test consists of a comparison of distance between empirical copula and an estimated copula under the null. This is the empirical process and it is given by:

$$C_n = \sqrt{n}\{C_n - C_{\theta_n}\} \qquad [5]$$

CvM GoF technique based on empirical process is:

$$S_n = \int_0^1 \{C_n(u)\}^2 \, dC_n(u) \qquad [6]$$

Large values of this test lead to rejection of H_0 . Analogously, Kendall process takes the form:

$$K_n = \sqrt{n}\{K_n - K_{\theta_n}\} \qquad [7]$$

Genest et al. (2006) proposed the GoF based on Kendall tau as follows:

$$S^K{}_n = \int_0^1 \{K_n(v)\}^2 \, dK_{\theta_n}(v) \qquad [8]$$

According to Berg (2007) and Genest & Rémillard (2008), a Monte Carlo procedure consists of repeating the test and sorting results and then determining, with 5% level, the percentage of rejection of the null hypothesis. Table 4 presents copula GoF results.

		Copula under Ha			
		Normal	**Student**	**Gumbel**	**Clayton**
Copula under H0	Normal	4.48%	39.81%	23.37%	24.67%
	Student	3.19%	5.17%	3.78%	5.40%
	Gumbel	34.06%	77.86%	4.85%	44.59%
	Clayton	38.35%	84.54%	27.67%	5.15%
Notes: Null hypothesis is Normal or Student or Clayton or Gumbel copula and alternative hypothesis comports same copulas. CvM test is repeated 10000 times with Monte Carlo simulation approach. Result of each iteration is 0 (the null is accepted) or 1 (the null is rejected). We compute					

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number of 0's from 10000 results which must be more than 95% to accept the null or less than 5% to reject the null.

Table 4: Percentage of rejection of null hypothesis in copula GoF

The calculated distance between empirical Kendall tau and the estimator under the null show that if Student copula is the null hypothesis, then all test values are low close to selected 5% level which suggests acceptance of the null. By cons, if the data are simulated from the Student copula, then the rejection of the null hypothesis is 39.81 % for the Normal copula, 77.86% for the Gumbel copula and 84.54 % for the Clayton copula. These percentages prove that best copula that better fits our data is Student copula with 10-dimension and 2 DoF.

5. Estimation of portfolio risk measures

After estimating and selecting right marginal distribution of each asset return series and estimating and selecting their true dependence structure, we can construct, respect to Sklar's theorem[4] a Multivariate Distribution with Copula (MVDC). From this MVDC we generate, with Monte Carlo approach, data and we estimate portfolio risk measure. We compose an equally weighted portfolio of ten assets and we estimate the quantile which means the VaR and average losses beyond the VaR which is the CVaR.

From Table 2 we couple different copulas to different marginals and we simulate 7822 random returns from each model. For each model, we estimate VaR and CVaR empirically. We repeat this simulation 10000 times and we estimate for each time VaR and CVaR. We calculate average VaR and average CVaR from iterations with probabilities 5%, 1% and 0.1%. Table 5 summarizes these risk measures.

Marginal distribution	Dependence structure			
	Clayton	Student	Normal	Gumbel
VaR 5% (Historical VaR 5% is -4.013%)				
Normal	-0,37%	-0,38%	-0,36%	-0,36%
Student	-131,24%	-118,49%	-113,49%	-118,93%
Skew Normal	-5,29%	-5,17%	-5,79%	-4,75%
Skew Student	-6,60%	-4,93%	-4,35%	-3,86%
VaR 1% (Historical VaR 1% is -8.821%				
Normal	-0,50%	-0,39%	-0,40%	-0,41%
Student	-191,88%	-182,43%	-173,65%	-125,23%
Skew Normal	-7,33%	-6,57%	-6,34%	-5,56%
Skew Student	-12,26%	-9,40%	-11,52%	-4,91%
VaR 0,1% (Historical VaR 0.1% is -13.304%)				
Normal	-0,55%	-0,41%	-0,50%	-0,42%
Student	-306,53%	-233,60%	-196,00%	-127,21%
Skew Normal	-7,67%	-7,88%	-6,97%	-5,62%
Skew Student	-33,54%	-23,85%	-19,28%	-6,33%
CVaR 0,1% (Historical CVaR 0.1% is -14.812%				
Normal	-0,55%	-0,42%	-0,51%	-0,42%
Student	-319,27%	-239,29%	-198,48%	-127,43%
Skew Normal	-7,71%	-8,03%	-7,04%	-5,63%
Skew Student	-35,90%	-38,78%	-20,15%	-6,48%

Table 5: Portfolio risk measure estimation

[4] This theorem suggests that if we have continuous marginals and a unique multivariate copula, then we can couple them in order to construct a joint multivariate distribution

Estimated risk measures empirically show that if probability if low, VaR is high. VaR with 5% probability is the half of the loss on the 99% quantile, which equals 8.821% of the value of the portfolio and CVaR with probability 0.1% is 14.812%. This first comparison proves the effectiveness of measuring risk with a coherent measure in the sens of Artzner et al. (1999).

From Table 5, the gap between risk measures estimation is high regards to marginals and copula used in simulation. For an MVDC of Normal margins and Normal copula, VaR with 5% probability level is -0.362% and for an MVDC of Student margins and Clayton copula, this measure is -131.238%, this means a gap of 130% approximately. This gap between risk measure estimation is high when precision of risk measure arises. for example, from VaR with 1% to VaR with 0.1% probability level, the gap between estimation is 318.911%.

Very low estimations are given by a model containing Normal Copula and Normal margins and very high estimations are from combination of Clayton copula and Student margins. This result is not surprisingly because characteristics of Normal distribution that can't take into account extreme values on tails and the Normal copula has no tail dependence. For model of Student margins with fat tails it captures extreme losses and the Clayton copula has a negative tail dependence.

To assess errors specification of marginal distribution or copulas on risk measure estimation we compare true model with other models. The difference is the effect of a wrong selection of best copula and/or best margins. If we use symmetric marginals and we assess the effect of a misspecification of dependence structure on risk measure estimation, we conclude that this effect is small for asymmetric copulas (underestimation of 70% in average) than symmetric copulas (overestimation of 43% in average).

If we use asymmetric dependence structure and we assess the effect of a misspecification of marginal distribution on risk measure estimation, we see that the effect is very high for Student margins (an overestimation of 215.744 %) than for Normal margins (an underestimation of -20.992 %).

6. Conclusion and discussion

Given that risk measures, VaR or CVaR, are estimated by portfolio joint distribution function. This JDF can be constructed by copula using Sklar's theorem by coupling marginal distributions to their dependence structure. The question is what's the impact of selecting the wrong copula and/or the wrong marginal distribution on risk measures estimation. We used marginal distributions, symmetric and asymmetric and different copulas function, symmetric and asymmetric in order to give a response to this question. GoF technique is used to select the true margins and true dependence structure and Monte Carlo simulation to simulate data from JDF. Data used in this paper, is financial asset returns and an equally weighted portfolio is constructed. Risk measures are VaR and CVaR with different probability levels. Best model selected with GoF is a JDF combination of Multivariate Student copula, as a dependence structure measure, and Skew Student margins.

Very low estimations are given by a model of Normal margins and Normal copula. But very high estimations are given by Clayton copula and Student margins. The difference is 320%, approximately. Our results proved that a wrong dependence structure has a lower effect approximately 34%) on risk measure estimation than a wrong marginals (approximately 170%). This is confirmed by Cheng, Li & Shi (2002),Ané & Kharoubi (2003). We proved that when we use lower probability levels for risk measures, the effect of a misspecification of marginals and/or copulas become very high. We had shown that Student copula has the highest effect on risk measures estimation, which corroborates with Fantazzinni (2009) results. We can improve these results if we use conditional distributions and conditional copulas or if we GARCH models and extreme value theory. In the same prolongation of this work, we can use a portfolio with optimal weights calculated by portfolio optimization problem or a portfolio of nonlinear financial products like options or Futures.

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